SUM RULE DESCRIPTION OF COLOR TRANSPARENCY

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ABSTRACT

The assumption that a small point-like configuration does not interact with nucleons leads to a new set of sum rules that are interpreted as models of the baryon-nucleon interaction. These models are rendered semi-realistic by requiring consistency with data for cross section fluctuations in proton-proton diffractive collisions.

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1. Introduction

Color Transparency (CT) is the suppression of final (and/or initial) state interactions caused by the cancellation of color fields of a system of closely separated quarks and gluons [1,2]. For example, in an (e,e'p) reaction with good kinematics the proton wave will be better approximated by a plane wave than by the usual exponentially decreasing wave function. The kinematics are "good" if the nuclear excitation energy is known well enough to ensure that no extra pions are created. With these kinematics the final state interaction is either elastic, or a two-body reaction. Such reactions are topics of active experimental investigation [3-5].

Color transparency CT occurs if: (1) a small-sized wave packet (point-like configuration or PLC) is formed in a high momentum transfer reaction [1,2], (2) the interaction between such a small object and nucleons is suppressed (color neutrality or color screening in perturbative QCD [6], the composite nature of hadrons in nonperturbative QCD [7]), (3) the wave packet escapes the nucleus while still small [8].

The evidence for item (2) is well-documented, so we take it as given. The effects of color neutrality (screening) in hard processes are reviewed in Refs [7,9]. Attempts to apply the two gluon exchange model to soft processes are discussed in Ref. [10].

The truth or falsehood of item (1) at experimentally feasible energies is an interesting issue discussed in Refs. [9,11,12]. Here we take the formation of a PLC as a starting point.

The escape process is our present concern. At infinitely large momentum transfer, CT is a consequence of quantum chromodynamics QCD, see e.g.Ref. [13]. For very large but finite energies, the cross sections, for processes we consider, decrease rapidly with momentum transfer. So, the feasibility of searching for color transparency becomes questionable, unless sub-asymptotic momentum transfer processes can be interpreted. For such kinematics, the PLC wave packet undergoes time evolution which increases its

size. The time τ to achieve characteristic hadronic size is $\tau \approx E/m\tau_0$, where $\tau_0 \approx 1$ fm is a characteristic rest frame time, and E is the lab energy. The mass m may be roughly the average of the masses of the nucleon and its first excitation according to Refs. [8,9,12,14,15]. So, for presently available energies, $\tau \sim 5\tau_0 \approx 5$ fm is small enough so that the PLC expands significantly as it moves through the nucleus. Thus the final state interaction is suppressed but does not disappear; its effects are studied in Refs. [8,9,14-15]. For example, the use of pQCD and Bjorken scaling for deep inelastic processes [8] allows one to obtain the dependence of PLC-nucleon cross section on the distance from the point of the hard interaction. Another procedure is to decompose the PLC as a coherent sum over hadronic states and to study the propagation of the PLC using this basis [14].

The purpose of this paper is to introduce a class of exactly solvable models for CT. In these models, the PLC (formed in a hard process) is treated as a coherent superposition of a finite number of hadronic states. We then assert that, at the point of formation, the PLC-baryon interaction vanishes, Eq. (7). Expressing the matrix elements of this vanishing interaction in a hadron basis and using completeness leads to a new set of sum rules yielding relationships between elastic and inelastic soft hadronic amplitudes. These relationships specify the form of the baryon-nucleon scattering matrix, so that one avoids assuming a specific interaction at the quark level. Such an idea has been applied first to CT in photoproduction of charmed particles [7]. See also Refs. [9,16].

These models yield new relations between soft and hard processes. Since a sufficiently hard amplitude can be calculated in pQCD, the present work may represent the beginning of a new method to calculate soft amplitudes: the generalization of dispersion sum rule approaches to the physics of scattering processes.

Our models involve the full transition matrix $\frac{-i}{2E}\widehat{T}$ for $XN \to X'N$ reactions, where X, X' represents any baryonic state. (To understand the factors, see below Eq. (11).)

A very large number of unmeasured matrix elements are then parameterized. We avoid an unnecessarily large number of parameters by imposing constraints based on existing data. In particular, using the recent analysis [17] of matrix elements of $\widehat{T}^n$ in high energy proton-proton diffractive processes leads to significant constraints. This makes the models more realistic than otherwise.

Here is an outline of the rest of the text. Some necessary formalism is recalled in Sect. 2. The new sum rules and models are introduced in Sect. 3. The models are constrained by diffractive dissociation data in Sect. 4. Results are presented in Sect. 5. A few summary remarks form Sect. 6.

2. Scattering Formalism

We begin the technical discussion by recalling some scattering formalism for relevant reactions. To be specific, we discuss the (e,e'p) reaction for quasielastic kinematics [4]. Other reactions could easily be considered using only modest extensions of our equations. The (e,e'p) reaction proceeds when a proton $|N\rangle$, bound in a shell model orbital α, absorbs a virtual photon. The absorption operator is the hard scattering operator $T_H(Q^2)$, and a PLC is formed:

$$|PLC\rangle = T_H(Q^2)\,|N\rangle. \tag{1}$$

The PLC is thus a superposition of (a generally infinite number of) hadron states having baryon number of a nucleon. In practice, we restrict ourselves by considering the contribution of either one or two resonances (or excited states) of the nucleon. Such an approximation is an effective method to treat convergent dispersion integrals (see e.g. Ref. [18]) and greatly simplifies the resulting models.

The amplitude for the proton knockout process is given by

$$\mathcal{M}_\alpha = \langle N, \alpha|\, T_H(Q^2)\, |\mathbf{\Psi}_{N,\vec{p}}\rangle, \tag{2a}$$

where $|\mathbf{\Psi}_{N,\vec{p}}\rangle$ is the scattering wave function. The subscripts N and $\vec{p}$ Refer to the boundary condition that the detected baryon is a proton of momentum $\vec{p}$. For quasielastic kinematics, $\vec{p}$ is essentially parallel to the direction of the virtual photon. The measured cross section is given by

$$\sigma \sim \sum_{\alpha,occ} |\mathcal{M}_\alpha|^2, \tag{2b}$$

where σ is obtained by integrating the angular distribution of the outgoing proton. The sum is over all occupied shells. We shall be concerned with ratios of σ to the corresponding quantity σ^B which is evaluated by using a plane wave function, instead of $|\mathbf{\Psi}_{N,\vec{p}}\rangle$, in Eq. (2a).

The coordinate space representation of $|\Psi_{N,\vec{p}}\rangle$ is a column vector $\Psi_{N,\vec{p}}(\vec{R})$ where $\vec{R}$ is the distance between the center of mass of the baryon and the nuclear center. The component of $\vec{R}$ parallel to $\vec{p}$ is denoted $\vec{Z}$. Then $\vec{R} = \vec{B} + \vec{Z}$ and $\vec{B}$ is the impact parameter. The different entries in the column vector Refer to the different baryonic components or channels (N^*, Δ etc.) of the scattering wave function. A baryonic basis is used, so we introduce an operator, $\widehat{V}$, to describe the baryon-nuclear interaction. The operator $\widehat{V}$ is obtained by summing the baryon-nucleon transition matrix (describing the production of baryon resonances) $\widehat{T}$ over the nucleons and taking the nuclear expectation value of the result. The effects of correlations between the nucleons in the nucleus are not important in the present context and are ignored. Then $\widehat{V}$ is proportional to the nuclear density, ρ, and $\widehat{V} = \frac{-i}{2E}\widehat{T}\rho$.

We use a generalized eikonal approximation to deduce the equation describing PLC propagation through the nucleus. Spin-dependent effects are ignored here, so we start with the relativistic Schröedinger equation and optical approximation:

$$(\sqrt{-\nabla^2 + \widehat{M^2}} + \widehat{V})|\Psi_{N,\vec{p}}\rangle = E|\Psi_{N,\vec{p}}\rangle, \qquad (3a)$$

where $\widehat{M^2}$ is the baryon mass operator squared, $E = \sqrt{p^2 + M^2}$ and M is the nucleon mass. As noted above, the potential $\widehat{V}$ is calculated using amplitudes for NN interactions leading to production of resonances, cf. Eq.(8). We will consider situations in which $p > 1 GeV/c$. Then Eq. (3a) may be simplified by multiplying both sides by the operator $\sqrt{-\nabla^2 + \widehat{M^2}}$ and using Eq. (3a) to eliminate terms proportional to the square root operator. This gives

$$(-\nabla^2 + \widehat{M^2})|\Psi_{N,\vec{p}}\rangle = (p^2 + M^2 - 2E\widehat{V})|\Psi_{N,\vec{p}}\rangle, \qquad (3b)$$

in which terms proportional to $(\widehat{V})^2$ and a gradient of $\widehat{V}$ are ignored. This is valid at high energies E and is consistent with the eikonal approximation.

The eikonal approximation to Eq. (3) is developed by defining a new column vector $\boldsymbol{\Phi}$

$$\boldsymbol{\Psi}_{N,\vec{p}}(\vec{R}) = e^{i\widehat{p}Z}\boldsymbol{\Phi}(\vec{R}), \tag{4}$$

where the operator $\widehat{p}$ acts in the space of baryons: $\widehat{p}^2 = p^2 + M^2 - \widehat{M}^2$. The notation has been simplified by ignoring the subscripts $N,\vec{p}$ for $\boldsymbol{\Phi}$. The use of Eq. (4) in Eq. (3b) leads to

$$2i\,\frac{\partial\boldsymbol{\Phi}(\vec{R})}{\partial Z} = \widehat{U}\,\boldsymbol{\Phi}(\vec{R}), \tag{5a}$$

where

$$\widehat{U} \equiv e^{-i\widehat{p}Z}\,\frac{2E}{\widehat{p}}\widehat{V}\,e^{i\widehat{p}Z}. \tag{5b}$$

The basic tools to compute a wide variety of nuclear reactions, suggested in [9], are illustrated in Eqs. (1-5). Some applications and a more detailed discussion of the formalism are given in Ref. [19].

3.Sum Rules and Models

Now we turn to the new models. The simplest is defined by describing the PLC as a superposition of two states

$$|PLC\rangle = \alpha|N\rangle + \beta|N^*\rangle. \tag{6}$$

In general, α and β are complex functions of Q^2 representing elastic and inelastic form factors. We shall see that only ratios enter in our evaluations, so we treat these functions as constants (In Ref. [14] , $\alpha = \beta$). Eq. (6) represents the small-sized wavepacket at the instant of formation. As the wave-packet moves through the nucleus each component acquires a different phase. Thus the size and other properties change. In our formalism, the effects of such phases appear in the exponential factors of Eq. (5b).

We determine the interaction $\widehat{U}$ by demanding that

$$\widehat{U}(\vec{B}, Z = 0)|PLC\rangle = 0. \tag{7a}$$

This relationship is equivalent to a set of sum rules. To see this, use completeness to express the PLC in terms of a set of hadronic states $\mid m\rangle$, and take the overlap with one of those, $\mid n\rangle$. This gives

$$\sum_m \langle n \mid \widehat{U}(\vec{B}, Z = 0) \mid m\rangle\langle m \mid PLC\rangle = 0. \tag{7b}$$

Thus one obtains a different sum rule for each state $\mid n\rangle$. These equations can be developed in different ways. One can take $\mid n\rangle$ to represent the nucleon and then break up the sum into the discrete nucleon term and a continuum for the other states[16]. Here we represent $\widehat{U}(\vec{R})$ as a two- or three- dimensional basis.

We start with the two-state basis in which the PLC is given by Eq. (6). Then the sum rules of Eq. (7) are given by an explicit expression:

$$\widehat{U}(\vec{B}, Z) = -i\sigma\rho(\vec{R}) \begin{pmatrix} 1 & -\frac{\alpha}{\beta}e^{i\Delta pZ} \\ -\frac{\alpha^*}{\beta^*}e^{-i\Delta pZ} & \mid \frac{\alpha}{\beta} \mid^2 \end{pmatrix}. \tag{8}$$

Here, $\Delta p = p_{N^*} - p$, $p_{N^*} = \sqrt{p^2 + M_N^2 - M_{N^*}^2}$. The real part of the nucleon-nucleon scattering amplitude is neglected here, as is valid at the high energies ($p > 1 GeV/c$) we consider. This is understood in terms of Regge theory: keeping vacuum pole exchanges, and neglecting secondary Regge poles is a good approximation. Thus the nucleon-nucleon scattering amplitude is proportional to the total cross section, $\sigma \approx$ 40 mb, so that one element of the $\widehat{U}$ matrix is determined by data. In addition, time-reversal invariance is used to determine the relationship between the off-diagonal matrix elements.

The resonance mass M_{N^*} and the related value of Δp controls the energy or momentum p for which color transparency occurs. If the value of M_{N^*} is small enough so that $\Delta p Z$ can be ignored, the interaction $\widehat{U}$ vanishes and color transparency occurs. We do not study the appropriate values of M_{N^*} here; the work of Ref. [16] argues that reasonable values of M_{N^*} are not very large $\approx 1.7 GeV$.

The interactions of $\widehat{U}$ of Eq. (8) vanish at Z=0, increase with Z to a maximum at $\Delta p Z = \pi$, and vanish again when $\Delta p Z = 2\pi$. Thus the size of the PLC changes as it moves through the nucleus.

We stress that, in this model, the ratios of the soft amplitudes for N^* to N production in nucleon-nucleon scattering are equal to ratios of hard form factors. This is an example of a non-trivial relationship between soft and hard processes that one expects to exist in QCD. In general, $\widehat{U}$ should act in a large space and these many states could complicate a more realistic treatment. It may be easier to examine the $c\bar{c}$ system $(J/\Psi, \Psi')$ [7] to discover these soft-hard relationships.

The reader may be puzzled by the relations between the soft matrix elements of the optical potential operator $\widehat{U}$ and the hard baryonic (transition) form factor matrix elements, α, β. It is therefore worthwhile to exemplify how a simple quark idea can lead to unexpected relationships between hadronic matrix elements. Consider Bjorken

scaling in deep inelastic scattering (DIS). As explained in many reviews, e.g. Refs. [7,9], at low x the DIS mechanism is that of virtual photon (γ^*) decay into a $q\bar{q}$ color singlet pair which interacts with the target nucleon. If the quark and anti-quark are closely separated, gluon emission effects are approximately cancelled (color screening CS). This CS is the essential element needed to obtain scaling at low x and is also necessary for CT to occur. Color screening is most simply expressed in terms of quarks and gluons, but a hadronic basis may also be used. The cross section for γ^*-nucleon scattering is given by :

$$\sigma(\gamma^*N) = \sum_{n,r} \frac{c_n \langle n \mid T^{SOFT} \mid r \rangle c_r^*}{(Q^2 + m_n^2)(Q^2 + m_r^2)}, \tag{9}$$

when expressed in terms of hadronic matrix elements. This equation describes the process of γ^* transition to a hadronic state n (of matrix element c_n); interaction with the target nucleon converts the state n to a state r (T^{SOFT}); followed by conversion to the γ^* (c_r^*).

The sum of the diagonal transitions is related to the γ^* polarization operator as

$$\Pi(Q^2) = \sum_n \frac{|c_n|^2}{(m_n^2 + Q^2)}, \tag{10}$$

which according to a QCD quark-loop calculation behaves as $\sim Q^2 ln Q^2$. The Bjorken scaling, which is well satisfied experimentally, requires $\sigma(\gamma^*N) \sim \frac{1}{Q^2}$. At the same time, comparing the expressions for $\sigma(\gamma^*N)$ and $\Pi(Q^2)$ shows that keeping only the diagonal contribution leads to $\sigma(\gamma^*N) \sim \frac{1}{Q^2}\Pi(Q^2) \quad \sim ln Q^2$ - the so-called Bjorken paradox. The quark ideas and experimental results are that this $ln Q^2$ term must vanish. This occurs only if

$$\sum_{n,r} c_n \langle n \mid T^{SOFT} \mid r \rangle c_r^* = 0. \tag{11}$$

This result as well as, for example, that of Eqs. (7) and (13) below would be very difficult to guess at using only hadronic ideas. (QCD logarithms are ignored for simplicity here.)

The above relation is an example, other than Eq. (7) of an unexpected relationship between hadronic matrix elements.

Thus perhaps Eq. (7) is not so surprising; and we shall proceed by evaluating its consequences. Before doing so, it is necessary to to be aware that two-state models are not realistic. One cannot model a wave packet of very small size as a coherent sum of only two eigenfunctions corresponding to normal-sized systems. For example, one can make $\langle r^2 \rangle$ vanish for a superposition of ground and first excited states bound in a given potential. But then $|\langle r^4 \rangle|$ would be rather large. Moreover, diffractive dissociation data [20] for the pp→pX reaction show that many states X (including a resonance region and a higher mass continuum) are excited.

We therefore make a three state model. *Each* of the proton, the resonance region and high mass region is represented as one state in this model. The derivation of the three-state model is as before. This PLC is defined as

$$| PLC_3 \rangle = \alpha \, | \, N \rangle + \beta \, | \, N^* \rangle + \gamma \, | \, N^{**} \rangle. \tag{12}$$

The soft operator $\widehat{U}_3$ is again defined so that $\widehat{U}_3(\vec{B}, Z = 0) \, | \, PLC_3 \rangle = 0$. This implies that

$$\widehat{U}_3(\vec{B}, Z = 0) = -i\sigma\rho \begin{pmatrix} 1 & -\frac{\alpha}{\beta} + \frac{\gamma}{\beta}\epsilon & -\epsilon \\ -\frac{\alpha^*}{\beta^*} + \frac{\gamma^*}{\beta^*}\epsilon^* & \mu & \frac{|\alpha|^2}{\beta^*\gamma} - \frac{\alpha\gamma^*}{\beta^*\gamma}\epsilon^* - \mu\frac{\beta}{\gamma} \\ -\epsilon^* & \frac{|\alpha|^2}{\beta\gamma^*} - \frac{\alpha^*\gamma}{\beta\gamma^*}\epsilon - \mu\frac{\beta^*}{\gamma^*} & \mu\frac{|\beta|^2}{|\gamma|^2} - \frac{|\alpha|^2}{|\gamma|^2} + 2\Re(\frac{\alpha}{\gamma}\epsilon^*) \end{pmatrix}. \tag{13}$$

In this three state model, the free parameters are taken to be the masses of the two excited states, M_{N^*} and $M_{N^{**}}$, the excited diagonal "optical factor" μ, the ratios $\frac{\alpha}{\gamma}$, $\frac{\beta}{\gamma}$ and, the coupling factor ϵ. The quantities μ, $\nu \equiv \mu\frac{|\beta|^2}{|\gamma|^2} - \frac{|\alpha|^2}{|\gamma|^2} + 2\Re(\frac{\alpha}{\gamma}\epsilon^*)$ are constrained to be greater than zero by unitarity. Further, at high energies it is well known that scattering amplitudes are predominantly imaginary. therefore, we choose our parameters to be purely real so that $\widehat{U}_3$ is purely imaginary.

The results of this model depend upon six $(\frac{\alpha}{\gamma}, | \frac{\beta}{\gamma} |, \epsilon, \mu, M_{N^*}, M_{N^{**}})$ parameters. The reader may wonder if this is too many. We note that the matrix of Eq. (13) is a model of the entire matrix of baryon-baryon scattering amplitudes. As such it represents an infinite number of matrix elements. Our point is that Eq. (7) provides a powerful and testable constraint on this matrix.

4. Contraints on model parameters

One may improve the model by imposing constraints which relate the parameters to soft process observables. The matrix of Eq. (13) is related to ratios of inelastic to elastic proton-nucleon cross sections. Making a detailed comparison between the parameters used here and data seems unrealistic. Instead we employ information regarding fluctuations of hadronic cross sections obtained recently by Blättel et al. [17]. Those authors used the Good-Walker [21] cross section eigenstate formalism to analyze inelastic shadowing and diffractive dissociation data in terms of moments of the transition matrix, $\langle N \mid \widehat{T}^n \mid N \rangle$ where

$$\widehat{T} \equiv e^{i\widehat{p}Z} \frac{i\widehat{U}}{\rho} e^{-i\widehat{p}Z}. \tag{14}$$

With a purely imaginary $\widehat{U}$, $\widehat{T}$ represents the imaginary part of the baryon-nucleon transition matrix; as such it is independent of $\vec{B}$ and $\vec{R}$. The results of Blättel et al. (for 200 GeV protons) are that

$$\langle N \mid \widehat{T}^2 \mid N \rangle = (1 + \omega_\sigma)\sigma^2, \tag{15a}$$

with $\omega_\sigma = 0.2$. Furthermore

$$\langle N \mid \widehat{T}^3 \mid N \rangle - \sigma \langle N \mid \widehat{T}^2 \mid N \rangle = \kappa_\sigma \sigma^3, \tag{15b}$$

where $\kappa_\sigma \sim 2\omega_\sigma$.

At lower energies, the relevant number of (parton) degrees of freedom is less than that at higher energies. Thus a smaller value of ω_σ is expected. Here we take

$$\omega_\sigma \lesssim 0.1, \qquad \kappa = 0.2. \tag{16}$$

These equations strongly constrain the matrices of Eqs. (8) and (13). Indeed, it only takes a moment to rule out all two-state models with purely imaginary amplitudes. For the two-state model of Eq. (8), the evaluations of Eq. (15) yield

$$1+ \mid \alpha/\beta \mid^2 = 1 + \omega_\sigma \tag{17a}$$

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and

$$(1+ \mid \alpha/\beta \mid^2)^2 - (1+ \mid \alpha/\beta \mid^2) = \kappa_\sigma \approx 2\omega_\sigma. \qquad (17b)$$

The use of (17a) in (17b) then leads to the result $\omega_\sigma = 1$. This failure of two-state models is consistent with the intuition that the expectation value of r^4 will be large even if that of r^2 vanishes. All of the results shown below satisfy the constraints of Eqs. (15) and (16).

It is also reasonable to see if the quantities β/α and γ/α represent ratios of excited state to proton form factors. In principle, these ratios are measurable. In the present model, $\mid N^{*,**}\rangle$ represents a coherent superposition of resonances, so that it is not possible to compare β/α and γ/α measured baryon resonance form factors. Below, we use values of β/α of the order of three or five. The measured ratios for single resonances are of the order of unity [22], so that our state $\beta \mid N^*\rangle$ is consistent with a superposition of a few resonances.

5. Results

We now turn to the evaluation of the model. Analytic solutions may be obtained by assuming that the PLC is produced at $Z = 0$ and propagates a distance L through uniform nuclear matter of constant density, $\rho = \rho_0 = 0.166 \ fm^{-3}$. For many sets of parameters of this three-state model, Eq. (5) reduces to three coupled first order differential equations, convertible into a third order equation with constant coefficients. The solution is an exponential. The eigen-'energies' can be found exactly, because a general cubic equation is soluble. The solution is useful for checking the numerics, but is not illuminating and omitted.

One may mimic the physics of the (e,e'p) reaction with this one-dimensional model by letting the distance L be the average distance across a nucleus, $L = 0.64 \, A^{1/3} \ fm$, and by setting $\vec{p}$ equal to the momentum of the virtual photon. The equations which result from taking a constant nuclear density are very similar to the well known molecular beam physics equations which lead to Rabi's formula [23]. We compute the probability that, after traversing a distance L through the nuclear matter, a nucleon (or an excitation) will be detected. Then the observables are the probabilities, $\mathcal{P}_N(L) = |\langle N|\Phi(L)\rangle|^2$. Similarly, the probability to produce an N^* or N^{**} is given by $\mathcal{P}_{N^{*(**)}}(L) = |\langle N^{*(**)}|\Phi(L)\rangle|^2$.

The results of the nuclear slab calculations are shown in Fig. 1. It is necessary to discuss how the parameter sets were chosen. We choose $M_{N^*} = 1.4 \, \text{GeV}$ and $M_{N^{**}} = 2.4$ GeV as representative of the resonance and continuum regions. We make no attempt here to fine-tune or constrain these mass values. It is clear that using larger values will lead to larger effects of CT. After the masses were chosen, α/γ was chosen to be either large (3), moderate (1), or small (1/3). Then the values of β/γ and ϵ are chosen over a broad grid of values. The values of μ and ν (third diagonal matrix element of Eq. (13)) are then uniquely determined by the constraints of of Eq. (16). Finally, only

those parameters sets leading to values of μ of the order of unity are kept. That term represents the ammount of absorption in the N^*-nucleon diagonal scattering, which we take to be of the same order as that of the proton-nucleon.

As shown in Fig. 1, the overall size of the probabilities, e.g. $\mathcal{P}_N(L)$, depends mainly on the importance of the second excited state (value of γ). Results with large, moderate, and small values of α/γ are shown in Fig. 1. All of the parameter sets discussed above lead to significant sizes of the probabilities. We call attention to the interesting oscillations (wiggles) as a function of the photon three-momentum, Figs. 1c and d. Furthermore, effects of constructive interference can lead to cross section ratios (transparencies) larger than unity (Fig. 1d), which may occur if ϵ is negative.

The next step is to generalize this one-dimensional calculation to that of the three-dimensional formalism of Eqs. (1-5). This is done by allowing the nuclear density ρ to be a function of $\vec{B}$ and Z. Then, within the eikonal approximation, the column vector $\boldsymbol{\Phi}$ ($| \boldsymbol{\Psi}_{N,\vec{p}}\rangle$) is known for that $\vec{B}$. The cross sections σ and σ^B are obtained by using the numerical solution for $\boldsymbol{\Psi}_{N,\vec{p}}$ (to compute σ) or plane wave functions (to compute σ^B) in the matrix elements of Eq. (2). The density $\rho(\vec{R})$ is described as a standard Wood-Saxon form. We also allow the masses of the baryon excitations to have a small imaginary part, 75 MeV. (This is not significant numerically, but simulates a typical resonance width of $\sim 150\ MeV$.) For details regarding the bound state wave functions and other aspects, see Ref. 14.

The use of the parameters of Fig. 1 lead to the very different "three-dimensional" results for the ratios σ/σ^B shown in Fig. 2. The wiggles are smoothed out, since the necessary average over impact parameters causes the path length to take on a continuous series of values.

The loss of the oscillations shows that averaging over impact parameters makes the expansion of the PLC more classical in the sense that effects of quantum mechanical

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interference are suppressed. One may search for the wiggles by considering processes in which the amplitude depends on higher powers of ρ. Then small impact parameters would be emphasized and the important values of the path length would be less spread out. One example, could be the production of backward particles in quasielastic processes. Another could be (p,2p) sub-processes in heavy ion collisions. This notion will be pursued in other work.

There are other noteworthy features in the comparison of Figs. 1 and 2. The ratios σ/σ^B are generally lower than the corresponding probabilities. This is caused by the diffuse nature of the nuclear surface. Even so, the ratios σ/σ^B are much larger than the standard Glauber calculation (keeping only the nucleon channel in the scattering wave function) labelled by DWBA in the Figures.

Another point of interest is that in Figs. 1a,b the apparent transparency of the second resonance is quite high, but this does not occur in Figs. 2a,b. For the parameter sets of those figures, ϵ is very small. Thus the production of the N^{**} is dominated by feeding from the N^*. This N^{**} production can only occur near the edge of the slab or (Fig. 1) or near the edge of the nucleus (Fig.2b) because the value of the second excited state diagonal element is very large. Indeed, in Fig. 1a (2a), $\nu = 130$ and in Fig. 1b (2b), $\nu = 14.2$. In contrast with the full nuclear matter density of the slab, the density and bound state wavefunctions are both quite small on the surface of the three-dimensional nuclear target. Thus the edge effects of N^{**} production through the N^* channel occur only for the nuclear slab.

The final result we show concerns using a set of parameters closely related to the model of Ref. [14]; see Fig. 3. As above, the solid curves denote the cross section ratio for a nucleon. The dashed and dotted curves represent the ratios for the first and second excited states in this three state model. The exponential approximations (EA and EA*) of Ref. [14] sum many higher order terms by exponentiating the first order (in $\widehat{U}$) result.

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The results here verify the accuracy of the exponential approximation, at least for the parameters of Fig.3. More than that, this result shows that a three dimensional baryon space is "large enough" to act as an infinite dimensional one for the model of Ref. [14]. Further details are discussed in Ref. [19].

Many sets of parameters lead to results qualitatively similar to those of Figs. 1-3. However, the use of a large value for the mass $M_{N^{**}}$ can suppress color transparency or push its appearance to a very high energy.

6. Summary

A new set of models of baryon-nucleon interactions is obtained from the assumption that a PLC does not interact with baryons. This involves new sum rules which relate hard and soft processes. The PLC's motion through the nucleus is governed by a solvable scattering wave equation. The model is made more realistic by requiring consistency with certain diffractive proton-proton scattering observables, Eqs.(12,13). These constraints allow significant effects of CT to occur. Furthermore, interesting effects such as rapid oscillations with energy and transparencies greater than unity are also allowed. Results are presented for the (e,e'p) and (e,e'N*) reactions, but the model can be applied [9] to any study of color transparency such as the (p,pp) reaction; the generalized optical potentials can be used in any process that requires nuclear scattering wave functions. For example, neutron-nucleus total cross sections can be computed. Another possible further extension could be to develop a new sum rule approach to learn about the properties of the PLC. One may evaluate matrix elements of $\widehat{T}^n$ in both the quark-gluon and hadronic bases. Since the two approaches must give the same result, powerful constraints can be obtained. Perturbative QCD or quark models can be used to get $\widehat{T}^n$ in the quark-gluon basis and some hadronic results can be obtained from data. Such new sum rules will be discussed elsewhere. Thus, the present work could lead to a new method to relate the results of pQCD to soft processes.

References and Footnotes

1. A.H. Mueller, "Proceedings of Seventeenth Rencontre de Moriond", Moriond, 1982 ed. J Tran Thanh Van (Editions Frontiers, Gif-sur-Yvette, France, 1982)p13.

2. S.J. Brodsky in Proceedings of the Thirteenth intl Symposium on Multiparticle Dynamics, ed. W. Kittel, W. Metzger and A. Stergiou (World Scientific, Singapore 1982,) p963.

3. A.S. Carroll et al Phys Rev Lett 61,1698 (1988; S. Heppelmann, p. 199 in "Nuclear physics on the Light Cone", ed. M.B. Johnson and L.S. Kisslinger , World Scientific (Singapore, 1989).

4. SLAC Expt. NE-18, R. Milner, Spokesman.

5. A.S. Carroll *et al.*, BNL expt. 850.

6. F.E. Low, Phys. Rev. D12, 163 (1975); S. Nussinov Phys. Rev. Lett 34, 1286 (1975); J. Gunion and D. Soper Phys Rev D15,2617 (1977).

7. L. Frankfurt and M. Strikman, Progress in Particle and Nuclear Physics, 27,135(1991); L. Frankurt and M. Strikman, Phys. Rep. 160, 235 (1988).

8. G.R. Farrar, H. Liu, L.L. Frankfurt & M.I. Strikman, Phys. Rev. Lett. 61 (1988) 686.

9. L Frankfurt, G.A. Miller & M. Strikman, "Color Transparency and Nuclear Phenomena", to be published Comm. Nuc. Part. Phys. Sept. 1992. 1992 UWA preprint-40427-26-N91.

10. B.Z. Kopeliovich, Sov.J. Part. Nucl. 21, 117 (1990).

11. H-n Li and G. Sterman,"The perturbative Pion Form Factor with Sudakov Suppression", 1992 preprint ITP-SB-92-10.

12. L. Frankfurt, G.A. Miller & M. Strikman, "Precocious dominance of Point-like configurations in Hadronic Form Factors", submitted to Nucl. Phys. A. U.Wa. preprint 40427-16-N92.

13. A.H.Mueller Phys.Rep.73,237 (1981)

14. B.K. Jennings and G.A. Miller, Phys. Lett. B236, 209 (1990); B.K. Jennings and G.A. Miller, Phys. Rev. D 44, 692 (1991); G.A. Miller and B.K. Jennings p. 25 in "Perspectives in Nuclear Physics at Intermediate Energies" Ed. S. Boffi, C. Ciofi degli Atti, M. Giannini, 1992, World Sci. Press Singapore; G.A. Miller, "Introduction to Color Transparency", in "Nucleon resonances and Nucleon Structure", G.A. Miller, editor. To be published by World Sci., Singapore (1992); B. K. Jennings and G.A. Miller, Phys. Lett. B 274,442 (1992).

15. B.Z. Kopeliovich and B.G. Zakharov, Phys. Lett. B264 (1991) 434.

16. B.K. Jennings and G.A. Miller, 1992 TRIUMF, U.Wa. preprint 40427-24-N92, "Realistic Hadronic Matrix Element Approach to Color Transparency" submitted to Phys. Rev. Lett.

17. B. Blättel , G. Baym ,L.L. Frankfurt, H. Heiselberg and M. Strikman, "Hadronic cross-section fluctuations", Univ. Illinois 1992 preprint, P92-6-66.

18. V. de Alfaro, S. Fubini,G. Furlan,and C. Rosetti "Currents in Hadrons" North-Holland (Amsterdam) 1973.

19. W.R. Greenberg and G.A. Miller "Multiple-Scattering Series for Color Transparency' UWA preprint 1992, 40427-23-N92, submitted to Phys Rev. D.

20. K. Goulianos, Phys. Rep. 101, 169 (1983)

21. M.L. Good and W.D. Walker, Phys. Rev. 120,1857 (1960).

22. P. Stoler, Phys. Rev. Lett. 66,1003 (1991); Phys. Rev. D44,73 (1991).

23. Rabi's formula is discussed in Chapt. 5 of "Modern Quantum Mechanics" by J.J. Sakurai, Addison Wesley, Redwood City, CA 1985.

Figure Captions

1. Nuclear slab probabilities, $^{12}C(e, e'p)$. The Solid- nucleon production. Dashed- N^* production. Dotted-N^{**} production. The parameters (μ, ν) (uniquely determined by ϵ, α/γ, and β/γ) for a,b,c, and d are (1.6, 130), (1.6, 14.2), (1.5, 1.2), (0.42, 0.96) respectively.

2. Three dimensional shell model ratios of cross sections. $^{12}C(e, e'p)$. Solid- nucleon production. Dashed- N^* production. Dotted-N^{**} production. DWBA-proton cross section ignoring CT effects is also shown as the straight solid line. The parameters μ and ν are as in Fig. 1.

3. Three dimensional shell model ratios of cross sections $^{12}C(e, e'p)$. The curves are defined in the text. See also Ref. [19].